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PERCEIVING INFINITY. DIFFERENT FACETS.

Methodological Articles

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Abstract

By means of a historical approach, we expound three different ways of perceiving the notion of infinity: mathematically, physically, and theologically. In the book of Genesis we are told of Man's attempt at reaching the skies, at becoming akin to the Gods. In Christian faith, down here is, at once, a place distant and much closer to its divine origin. He cannot be circumscribed. He is like an endless ocean of infinity. Furthermore, we use examples to emphasise how this notion is perceived by children and adolescents in the education system. The inferences at the end of this article underline how important it is for the tutor to understand the fascinating notion of infinity, and to find a useful place for it, inside the structure of the classes that tutor teaches.

1.The Notion Of Infinity

A) Mathematical Infinity

In 1932, mathematician Herman Weyl stated that *Mathematics Is The Science Of The Infinite*. The concept of infinity has been the object of many investigations, ever since ancient times. Amongst the philosophers to study it was **Zeno** of Elea (490 – 430 B.C.). He was a member of the Eleatic school and a disciple of **Parmenides**. Aristotle regards Zeno as the founder of dialectics. Zeno is widely known today through his paradoxes, the most famous of which are: Arrow's Paradox (also known as Fletcher's Paradox) and Achilles and the Tortoise. The notion of infinity and the method of transition to the limit can both be traced back to ancient times, and, more specifically, to ancient Greek mathematicians. Medieval scholastics were the first to come across such paradoxes, followed by the architects of infinitesimal calculus (**I. Newton and G. W. Leibniz**). The fact that the whole is greater than one of its parts, for instance, had been known since ancient times. Thus, they couldn't explain why $2\mathbb{N} = \{2k \mid k \in \mathbb{N}\} \subset \mathbb{N}$ and yet $2k \to k$ is, still, a *one-to-one correspondence* (or bijection) between the proper subset $2\mathbb{N}$ and the entire set $\mathbb{N}$, both of them *having the same number of elements*. Over the centuries, infinity has been regarded and analyzed as potential infinity. This is to say that after each element there comes another, creating, as with natural numbers, an endless sequence.

An interesting study on infinity was made by Albert Ricmerstrop (1316-1390), one of the most prominent logicians of the Middle Ages. Albert was educated in Prague and Paris. He later became the first rector of the University of Vienna, in 1365. In his book, *Sophismata*, he gives an elegant example of an infinite set placed in a bijective correspondence with one of its proper subsets, thus dispelling the Aristotelian idea that such a set cannot be. Albert's example suggests an infinitely long wooden beam, which is then cut into identical cubes, the sides of which have a length of one. The cubes are then placed in such fashion as to fill out the entire space. He places one cube and then surrounds it with $3^3 - 1^3$ other cubes. The side of the larger cube thus obtained has a length of 3 (since it is made up of a number of 3^3 initial cubes). It, in turn is then surrounded with $5^3 - 3^3$ more cubes, and the process continues.

Space is thus being filled up by use of a bijective correspondence between a set and one of its proper subsets.

Mathematician **Bernard Bolzano** (1781-1848) estimated that all infinite sets are cardinally equivalent. He laid out the bijective correspondence between the points on a semi-circle and those on a straight line.

We have **Georg Cantor** (1845-1918) to thank for having introduced the concept of actual infinity to the field of mathematics, by using the notion of bijective function. He was the one to discover that infinite sets do not have the same cardinality, and to create an arithmetic of transfinite numbers.

No one shall expel us from the Paradise that Cantor has created (**D.Hilbert**).

The theory of combinations deals with counting the elements of finite sets. The importance of solving counting problems relates to many a concrete application in different fields of knowledge. Using a finite, or, what is more, a minimum number of steps in solving an exercise is the cornerstone of developing algorithms in computer science. **Xenocrates** of Chalcedon (396-314 B.C) was one ancient philosopher and mathematician to solve difficult problems involving combinations and permutations. The notion of permutation was first introduced in an ancient, mystical manuscript, written somewhere between the year 200 and 600 A.D. by **Sefer Yetzirah**. By analysing ancient languages, such as Sanskrit, Chinese, Hebrew, Tibetan, Greek and others, one notices that Hebrew, alone, bears the first expression of language and mathematics through spirituality. Kabbalah stands as the language of the inner universe. It is still studied today, in search of new ways of understanding and deciphering the ancient texts.

Mathematician **John Wallis**, famous for developing secret codes during the English Civil War, took the Ancient Roman symbol $\subset|\supset$, used to represent the number 1,000, (though sometimes represented through **M**), and fashioned it into ∞. Since the number 1,000 was too large for casual, everyday use at the time, the symbol would, as of 1655, be used by mathematicians to represent *infinity*. There are other symbols associated with infinity: **Jacob Bernoulli's** lemniscate (1696), **St. Boniface's** cross (700) or the Ouroboros (1600 B.C.), depicting a snake eating its own tail.

The story of Hotel Infinity was used as a way of perceiving both the notion of countable set itself and the early mathematical operations involving such sets. The great mathematician **David Hilbert** was the one to come up with the story. A set (*A*) is only countable if there is a bijective function $f : \mathbb{N}^* \to A$. This means its elements can be listed as a sequence $A = \{a_1, a_2, ..., a_n, ...\}$.

The Story of Hotel Infinity. Somewhere in the infinite vastness of space, there is an unusual hotel with an infinite number of rooms. The rooms are numbered by means of every natural number. In order to stay at this hotel, the requirement is that only one person may occupy a room at any given time (for the sake of the tourists' comfort). We assume the hotel is fully booked, and so no rooms are available. From outer space there comes a person looking for accommodation. How can we provide lodging? We know that we cannot accommodate all tourists in a hotel with a finite number of rooms. Yet, in this unusual Hotel, we can. We give room number 1 to the new-comer, we move the person from room 1 to room 2, the one from room 2 to room 3... the one from room *n* to room *n*+1, and thus we accommodate everyone. Next, there arrive at the hotel a number of $m \in \mathbb{N}^*$ tourists, looking for accommodation. We know we find it easy to receive all of them. All the new-comers take the first *m* rooms, and everybody else gets moved in this fashion: the person from room n takes the room $n+m$. What can be inferred from this process is put forth in the following statement: *The union between a countable set and a finite set is countable*. Now that the hotel is booked, we have another infinite, countable set of tourists demanding accommodation. How can we provide lodging? The idea is this: we move all of the old tourists to rooms that have even numbers, and we accommodate the new ones in rooms that have uneven numbers. We can conclude that *The union of two countable sets is countable*. Since $-\mathbb{N} = \{-n | n \in \mathbb{N}\}$ is countable, the previous statement proves to us that $\mathbb{Z} = \mathbb{N}^* \bigcup (-\mathbb{N})$ is countable. By means of an infinite matrix, it can be demonstrated that the Hotel is able to accommodate a countable, infinite set of countable sets of tourists. Which is to say that *The countable union of multiple countable unions is, itself, countable*.

Cardinal numbers. Two sets (*A*,*B*) are cardinally equivalent. We say that $A \sim B$ if there is a bijective function $f: A \to B$. Cantor defines $|A| = \{B | B \sim A\}$, assigning, by extention, the cardinals of finite sets (which are natural numbers) to infinite sets, as well. If we take any two given cardinal numbers $\alpha = |A|$, $\beta = |B|$, we can say that $\alpha \le \beta$ if there is an injective function $g: A \to B$. Cantor also introduces the mathematical operations of addition, subtraction, multiplication and exponentiation of cardinal numbers.

Consider the sequence of countable sets $A_n = \left\{\frac{m}{n} | m \in \mathbb{N}\right\}, \forall n \in \mathbb{N}^*$. This means $\mathbb{Q}_+ = \bigcup_{n \ge 1} A_n$ is countable. From this we infer that $\mathbb{Q} = \mathbb{Q}_+ \bigcup \mathbb{Q}_-$ is countable. All sets that are cardinally equivalent to $\mathbb{N}^*$ are countable, and have the cardinal number $\aleph_0$. By employing of diagonalization argument, Cantor reveals, in 1873, that $\mathbb{R}$ is uncountable, since its cardinality is $c = 2^{\aleph_0}$, which is to say, a cardinality of the continuum.

He then goes on to prove the theorem bearing his name, specifically the one stating that any set (*A*) cannot be cardinally equivalent to $P(A)$, whence we arrive to the conclusion that $|A| < |P(A)|$. Thus he proves that there is an infinite number of transfinite numbers (cardinalities of infinite sets). Theologians took an interest in Cantor's theory, which proved that the sequence of transfinite numbers is not finite, therefore there can be no highest transfinite number.

Following the example of Liouville, from 1844, Cantor proves the existence of an uncountable, infinite set of transcendental numbers.

Following Cantor's development of the set theory, there have arisen certain contradictions. This leads to a certain amount of diffidence when defining sets and ultimately resulted in the creation of an axiomatic system in the matter. We also mention **Bertrand Russell**'s (1872-1970) paradox. Consider a given set. It is an ordinary set if it does not contain itself, as an element. Now consider *C* the set of all ordinary sets. Is this set ordinary or extraordinary? If *C* is ordinary, then one of its elements must be *C*, which means it's extraordinary, since it contains itself. Thus we arrive at a contradiction. If *C* is extraordinary, then it contains *C* as an element, which means *C* is ordinary (since *C* only contains ordinary sets). Again, we arrive at a contradiction. The premises of the continuum states that there is no cardinal number between $\aleph_0 = |\mathbb{N}|$ and $= |\mathbb{R}|$.

Kurt Gödel (1906-1978) demonstrated that, if the underlying axioms of set theory are consistent, then the extended axiom system (obtained through the addition of the continuum hypotheses) is also consistent.

B) Physical Infinity

Upon mentioning infinity one thinks of immensity – the stars, the galaxies, the vastness of space – but there is an inner, infinite space closer to home. Ever smaller objects may be obtained through repeated division in half, but there is a practical barrier.

The observable universe contains an approximate number of 10^{80} atoms.

Zeno (490 – 430 B.C.), being a student of Parmenide's, posited that movement is not possible.

Zeno's first paradox stated that, in order to cover a distance of 1 km, one must first cover half of that distance, namely $\frac{1}{2}$ km, then half of the remaining distance, and so on. However small, a fraction of the distance would always be left that one has yet to cover, which implies that one never reaches his destination. Thus, the distance covered will always look like this:

$$\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^n}<1,\ \forall n\in \mathbb{N}^*.$$

The second paradox is that of *Achiles and the Tortoise*.

The sky is dark because the Universe is old and vast. It expands by billions of light years at a time and has a very low density ($\approx 1 atom/1m^3$). If the Universe had a finite volume, the ever-expanding Universe theory would become null and void. The maximum finite speed at which all matter and information can travel is the speed of light in a perfectly void space. It is possible for a minimum interval and a minimum distance to exist, as dictated by natural constants. They are combinations of the speed of light, Newton's gravitational constant and Planck's quantum constant (the minimum length being of approximately 10^{-33} cm, and the minimum time of 10^{-43} s). By exceeding these values, nature suffers a quantum gravitational alteration, the nature of which is not yet fully understood. In modern cosmology, the physical universe can be regarded as finite.

Aristotle thought the material universe to be finite, but surrounded by an infinite void. Since the Earth was considered to be at the Centre of the Universe, it could not be infinite, because that would mean it had more than one centre. Aristotle distinguished between *potential* infinities and *actual* infinities. In his *The Physics*, Aristotle states: *Infinity has a potential existence... There is no actual infinity*, which is to say no real infinity. An example of a potential infinity is the sequence of natural numbers.

Thomas Digges (1546-1595) was the first Renaissance astronomer and scientist to consider the Universe's infinity. Giordano Bruno (1548-1600) reckoned there is only one, single universe, and that it is boundless and contains an infinity of stars and planets.

As far as Einstein is concerned, the mass and motion of objects determine the shape of space and the flow of time in different places. Away from the presence of objects with mass, space is almost flat and unaffected by them. If a small volume of space contains large amounts of masses and objects are moving at a speed close to that of light, then space-time distortions are significant. In these circumstances, if a physical quantity reaches an infinite value (say the temperature or acceleration of a given object), then the space-time curvature becomes infinite, which is to say that space is torn apart. Peter Bergmann, with whom Einstein had a close collaboration, wrote: *Singularities in classical field theory are intolerable. They are intolerable from the point of view of classical field theory because a singular region represents a breakdown of the postulated laws of nature.* The laws of gravity will cease to apply to the infinities of physics.

If a cloud of matter roughly three times the size of the Sun contracts under its own gravity, it will become invisible for an outside observer, once it reaches a certain critical size. It will have become a *black hole*. These can be massive, whilst having even a lower density than air itself. Even if we can admit the existence of an actual physical infinity inside the black hole, that physical infinity will still be masked by the event horizon. Thus, Roger Penrose argues, works the principle of *cosmic censorship*. This statement is regarded as being valid only under certain circumstances that are not to be found in real life.

C) Theological Infinity

Immortality is regarded as *the bravest gesture of our humanity towards the unknown*.

Faith encourages the hope that things unfold perpetually: *For ever and ever, Amen.*

In the book of Genesis we are told of Man's attempt at reaching the skies, at becoming akin to the Gods, by constructing the Tower of Babylon. Since the result of the builders' attempt was a far cry from their expectations, Man's search has ever since been regarded as a hopeless endeavour.

Plotinus (205-270 A.D.) regarded *One* as being infinite, at once endless plenitude and endless simplicity, absolute power and absolute order. *One* is, for Plotin, the unascertainable, yet benign source of all human virtues. It possesses said virtues through its indefinite, infinite state of perfection and simplicity. Infinity, as Plotinus sees it, is merely the basis for finity, precisely on account of its inability of ever being finite. Plotinus attempted to reconcile exacting reason with mystical aspiration.

Gregory of Nyssa (335-395) considers God's infinity to be love that gives of itself, freely. In Christian faith, *down here* is, at once, a place distant and much closer to its divine origin. He cannot be circumscribed. He is like an endless ocean of infinity.

The ontological, insurmountable difference between creation and God – between the dynamics of finitude and an infinity that is eternally dynamic – is, at once, infinity interfering in that which is finite, and finitude partaking in that which does not belong to it, but inside which it moves. It does so not dialectically, abstractly or merely theoretically, but through its own endless growth inside God's grace.

The perfect example of a virtuous soul is Moses. Though full to the brim with God's beauty, Moses is always longing for more. Not so much because of his own qualities, but on the account of God's actual being.

Theologian **David Bentley Hart** said, in an interview taken on January 10, 2006, that God is *in and beyond all things, nearer to the essence of every creature than that creature itself, and infinitely outside the grasp of all finite things.*

Christian tradition maintains that contemplating beauty and contemplating infinity coincide, in an unique manner. The original unity is the original *mutuality*.

As threefold love, God is the complete plenitude of an infinite dynamics, the absolutely complete and plenary germination of the Son and the proceeding of the Holy Spirit from the Father. Such is the infinite *drama* of the fortunate act of God's overflowing with God – which is His godly being. His love is infinite, ontological peace, not merely a metaphysical armistice. They are infinitely determined as the living love of divine persons *for one another*. We will cite Augustine: *In Trinity is the supreme source of all things, and the most perfect beauty, and the most blessed delight. Those three, therefore, both seem to be mutually determined to each other, and are in themselves infinite*. (De Trinitate 6.10.12)

God's spirit is also a light that shines on humanity. It shows that the isolated self is but a shadow, the faded mark on a forgotten gift that was offered before the self was a self. The spirit gives life to all life, in an endless diversity. It is believed that the Spirit is the light through which the Son can be seen, and the Father can be seen in the Son. He is the light of creation and the glow of Word's splendour that allows us to see the depths of the Father in the beauty of the Son and in the innermost body of the Son.

Christianity advocates for the coherence of totality, the music of unity, the infinite music of the three persons that give and receive and give again.

He is not the above that stands against the below, but the perpetual act of interspacing that gives the above and the below their own place. God is infinite as a Trinity, not only transcending all borders, but venturing beyond any border.

Georg Cantor said that God has inoculated the concept of number, both finite and infinite, into the mind of Man, in order to make him reflect upon His own plenitude.

2. Mathematical Examples That Endeavour To Define The Notion Of Infinity.

Kids as young as 3 years old look at the night sky, spangled with sparkling heavenly objects. Fascinated by them, they ask: *How far is it from here to there?, If there is a lot of them, how many are there?*, *Is there a border to the cosmos?.*

Similarly, when we are at the seaside, we get assaulted with questions like *How big is the sea?*, *How many fish are in the sea?*, *How many grains of sand are there on the beach*, *How big is the Earth?*.

Once they've learned how to count, even before first grade, children notice, by writing consecutive natural numbers, that they *never end*. Then, eventually, they doze off and take a nap (since the positional or decimal system of writing, discovered by the Indians and passed on to us by the Arabs, is wonderfully economical and eloquent).

In the fifth grade they learn that the sequence of natural numbers is infinite and that the sequence of prime numbers is infinite (theorem that was proven by **Euclid**). During middle school, they study the sets of numbers $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ and, from the inclusions $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$, they deduce that $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are infinite, since $\mathbb{N}$ is infinite (any set that includes an infinite set is itself an infinite set).

During basic geometry classes, middle school students learn that any straight line, any plane and space itself are infinite. Even if it is finite, a straight line segment is made up of an infinity of points. The more one increases the number of sides on an regular polygon that is inscribed in a circle, the more it brings the value of its perimeter ever closer to that of the length of the circle. The same goes for their respective surfaces. **Nicholas of Cusa** believed that *The intellect is to truth what [an inscribed] polygon is to [the inscribing] circle. The more angles the inscribed polygon has, the more similar it is to the circle.*

Also studied in middle school are **Zeno's** paradoxes, either during physics or mathematics classes.

In high school years, after having studied bijective functions, students take their first look at **G. Cantor's** theory of cardinal numbers. Scientific curiosity is augmented by means of **David Hilbert's** *Story of Hotel Infinity*. Any infinite set contains, at the very least, one countable set. Therefore the lowest transfinite number is $\aleph_0$. From Cantor's theorem we infer that there is an infinity of transfinite numbers, thus an infinity of different types of infinite sets. This is to say that *there are more* infinite sets of certain types than there are of others. As far as mathematical analysis is concerned, this applies to exercises concerning density, functional equations, continuity, the Darboux property, integrability, etc. In algebra, there are direct applications for establishing that two algebraic structures are non isomorphic, when subjacent sets have a different cardinality. We cite

D. Hilbert's words: *In a certain sense, mathematical analysis is a symphony of the infinite.* Examples of classes of functions can be produced, the cardinality of which demands determining. As for geometry, René Descartes's bijection creates the correspondence between any point on a straight line and the set of real numbers.

This correspondence leads to a whole new way of studying geometry, namely with the help of algebra, called analytical geometry. Renaissance thus united the purely geometrical spirit of the Greek with the algebraic spirit of the Arab.

There are just as many points on a (non-degenerate) straight line segment, as there are in a straight line, in a plane, or in space. All this is perceptible, then comprehensible for the high school student.

Switching from two dimensions to three, though illusively bringing *more freedom*, can sometimes be restrictive. In the plane, for instance, there can be an infinity of regular polygons, while in space there can only be five regular polyhedrons (platonic solids): the tetrahedron, the cube or hexahedron, the octahedron, the dodecahedron and the icosahedron.

These are basic notions that are quickly covered in college, during classes of mathematical analysis. Obviously, for those inclined, there are specialized treatises concerned with the theory of cardinal and ordinal numbers.

Thinking in probabilities, as **Blaise Pascal** (1623-1662) did – French mathematician and one of the founding fathers of the theory of probability – one can see that it is better to believe in God than not to. Here is Pascal's reasoning, as presented in his work, *Pensées*:

	God exists	*God does not exist*
One believes in God	infinite *gain*	*finite loss*
One does not believe in God	infinite *loss*	*finite gain*

If you are one to believe in God, either the gain is infinite, or the loss is finite, since He can either exist or not. If you are not one to believe in God, you may expect either an infinite loss or a finite gain, according to whether He exists or not.

Mathematicians have always been drawn to endless sequences. French mathematician **Nicole Oresme** (1323-1382), for instance, demonstrated in 1350, the divergence of the harmonic series $\sum_{n\geq 1}\frac{1}{n}$. Its divergence is, once again, attested in 1647, by Italian mathematician **Pietro Mengoli**, then again, in 1687, by Swiss mathematician **Johann Bernoulli**. Consequently, certain paradoxes concerning divergent series were discovered, such as the series $\sum_{n\geq 1}\left(-1\right)^{n+1}$, from which **G.Grandi** deduced, in 1710, that 0 = 1.

The notion of convergent series only achieved a clear contour through the efforts of **Henrik Niels Abel** (1802-1829) and of **Augustin-Louis Cauchy** (1789-1859).

3. Inferences

Studying infinity under these three different facets incites to further reading, creates a basis for coherent dialogue with our own disciples. We mention, hereinafter, a few educational aspects (picked out of an *infinite* amount) that may ensue from minutely observing the subject at hand.

- Emphasising the congruence between beauty, truth and kindness, so as to preserve the autonomy of aesthetics.
- In our current age, those partial to beauty have to bear the pains of exile. This is why we take the liberty of cultivating beauty. Beauty is the essence of theology. It is the extent and ratio of peace. Peace is the truth of beauty. Through perpetuation, the soul becomes one with eternity.
- Converting the soul to ponder on beauty. Change is a means that provides us the freedom to recover the extent of divine harmony.
- Historical incursions use natural curiosity to develop cultural memory. They reveal the views of the ancients on natural order. The past forever accompanies the present.
- Attempting to dim the schism between faith and reason.
- The joy of seeing the world through the eyes of our own disciples.
- Talking about God or a god means leading the spirit to the transcendent origin of being, or building our own myth.
- Self sacrifice, forth comingness towards the other (the disciple), these elucidate how the notions of *tutor*, *mentor*, *teacher* and *professor* can become equivalent.
- In ancient Islamic art, the mosaic used on flat or curved surfaces was a means to explore all mathematical symmetries. Repetition suggests infinity. Repetitive imagery generates fractals, by changing scales.
- Our brain allows us neural configurations of $10^{7\cdot 10^{13}}$ thoughts. These stem from making combinations from approximately 10^{27} atoms.

- Telling the *Story of Hotel Infinity* means intellectually challenging our students. It brings mathematics and literature together. It creates the desire for future connections between different finite sets, which are necessary as a prime example of algebraic factoring. Since partitions and equivalence relations are associated in a bijective manner, we mention that C. F. Gauss was the one to introduce the model on congruence. He also anticipated the equivalence relations, but did not name them as such. Ideas subjacent to the notion of equivalence relation are to be found in the treatise *I Principi di Geometria* (1889), written by Italian mathematician **Giuseppe Peano** (1858-1932). Applications of factoring finite sets were developed by 20th century mathematicians.

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