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BENFORD'S LAW IN THE CASE OF HUNGARIAN WHOLE-SALE TRADE SECTOR

Research paper

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Abstract

Benford's law has attracted many researchers for detecting the fraudulent data and can be used as one of the digital analysis tools for auditing of the accounting data. In this treatise, the accuracy of figures reported in Hungarian Trading Companies' data are examined through digital analysis technique with the consideration of Benford's Law. The net sales data from the period of year 2009 to 2014 has been used for detecting the anomalies and to confirm whether the digit-pattern follows Benford's distribution. Through the obtained results we claimed that the frequencies of first and second digits' place follow the Benford's theoretical distribution and exhibits to close conformity. Moreover analysis of the second, first-order and second-order gave a mixed result of close conformity to significant deviation from expected frequency. Also the absolute deviation (MAD) value of first and second digit suggest an overall conformity of the data to Benford's distribution.

INTRODUCTION

Financial statements data that is free from errors are crucial to effective allocation of capital and hence the accurate functioning of capital markets. The errors, biases and intentional manipulation of financial data hinders the accurate decision making process of the firm. Therefore, auditors, analyst and investors are more focused in finding out the ways to detect these errors (Amiram, Bozanic, & Rouen, 2015). Much of the attention is diverted to this issue after the corporate scandals in the start of last decade (Enron, Worldcom). These giant financial reporting failure resulted into decreased investors' confidence on quality of reporting and auditing (Rezaee, 2005). The increasing frequency of these cases have resulted into loss of billion dollars (ACFE, 2016). Prior literature provides us a number of different methods in assessing the deviation in financial statements (Dechow, Myers, & Walton, 2009; Owens, Wu, & Zimmerman, 2013; Debreceny & Gray, 2010; Gray & Debreceny, 2014)

Digital analysis is studying and understanding of pattern of digits in numbers (Nigrini & Mittermaier, 1997)A particular digital analysis technique is the analysis of position of digits and frequency in random set of numbers (Newcomb, 1881), later became Benford's Law after the name of its discoverer (Benford's, 1938). Benford's law has been used extensively to understand the discrepancies and misbehavior in data related to different fields such as economics, physics or accounting. Over past six decades, more than 200 different researches have been published about Benford's law, that is based on possibility of frequency of a particular digit at a particular position in number (Nigrini, 1999). These researches had advanced the use of this law (digital analysis) as an effective tool to identify unintentional errors as well as frauds (Durtschi, Hillison, & Pacini, 2004). References on the application of Benford's law in auditing include Carslaw, (1988), Nigrini (1994), Nigrini and Mittermaier, (1997), Drake & Nigrini, (2000), Durtschi et al., (2004), Diekmann, (2007), Watrin, Struffert, and Ullmann, (2008), Coderre (2009) and da Silva and Carreira, (2013).

The objective of this paper is to make an effective use of Benford's law to check the quality of reporting of Hungarian trading firms and to analyze whether or not the data shows conformity to Benford's theoretical distribution. We begin by analyzing the digit patterns for first digit, second digit and first two digits. Using Nigrini's template (Nigrini, 2012), a comparison of actual and Benford's distribution is made. Z-stat and Mean Average Deviation of digits distribution are analyzed to check robustness of result.

BRIEF REVIEW OF LITERATURE

Simon Newcomb a mathematician in the late 19th century published his article that later transformed to what we know as Benford's Law. Newcomb (p. 40) noticed that not all digits at first place appear with same pattern. Although he could not offer a theoretical explanation to this observation, his suggested expression for distribution of first number is

$$P(D_1 = d_1) = log\left(1 + \frac{1}{d1}\right) for\ d_1 \in \{1, \ldots, 9\} \quad (1)$$

Benford's gave an empirical testing to Newcomb's proposition. By using dataset of varying nature e.g., areas of river, addresses and population data, he confirmed Newcomb's hypothesis and explained the pattern of leading digit frequency (Benford, 1938). According to him, unbiased set of numbers will follow different proportion of first to nine digits at first position. Digit one will occur 30.1% of times, digit two will occur 17.6% of times and hence percentage will continue decreasing as leading digit reaches nine. This law is also termed as Law of First Digit.

$$P(D_2 = d_2) = \sum_{d1=1}^{9} log\left(1 + \frac{1}{d1d2}\right)\ for\ \ d_2 \in \{1, \ldots, 9\} \quad (2)$$

$$P(D_1D_2 = d_1d_2) = log\left(1 + \frac{1}{d_1d_2}\right)\ for\ d_1\ d_2 \in \{1, \ldots, 9\} \quad (3)$$

Where P indicates the probability of occurrence of event in parenthesis (Nigrini, 1996).

Nigrini (1996) provides a further discussion by analyzing taxpayer behavior and thus providing a solid foundation to use of this law for accounting data. He hypothesized that the accounting data reported truly must follow Benford's pattern of distribution, thus confirming the usefulness of this law to check the possible deviation in reported figures.

The other references to the use of this Law for analyzing accounting data include Carslaw, (1988) Nigrini, (1994), Nigrini & Mittermaier, (1997), Drake & Nigrini, (2000), Durtschi et al., (2004), Diekmann, 2007), Watrin et al., (2008), Coderre (2009) and (da Silva & Carreira, 2013). Carslaw, (1988) gave a more focus to second digit of reported income to detect income manipulation thus adding first accounting application of Benford's Law to literature. By using income number of companies from New Zealand, he performed a comparison of expected Benford's frequency and actual frequency. Durtschi et al., (2004) discussed type of frauds and the data for which Benford's law can be applied.

In order to check conformity of data, several conformity tests can be used. Nigrini & Mittermaier (1997) used six digital analysis tests, first digit test, second digit test, first two digit test, the number duplication test, the rounding test and the last-two digits test. To test conformity, chi square statistics, Mean Absolute Deviation (MAD), and normally distributed Z-statistic was applied.

Using the same insight (Nigrini & Mittermaier, 1997), this study is more focused towards analyzing the digital pattern and checking the conformity on the basis of Z-statistics and MAD statistic of to check the actual proportion of digits in net sales data and compare it to expected proportion. Particularly, Nigrini's template (Nigrini, 2012) is used for assessing proportion of different digits. A digit can be referred to 1 to 9 or it can be 45 because it shows first two digit combinations. Z stats take into account absolute difference between actual distribution and expected distribution.

$$Z = \frac{|P_o - P_e| - \left(\frac{1}{2N}\right)}{\sqrt{\frac{P_e(1-P_e)}{N}}} \quad (4)$$

Where P_o is actual proportion of test sample, P_e is expected Benford's proportion, N is the number of observation. The expression $\frac{1}{2N}$ is a correction term and is used only if it is smaller than the absolute difference between actual and expected term. The results of Z-test become more sensitive to the deviation as value of N rises (Nigrini, 2012). In order to overcome this problem Mean Absolute Deviation test is applied since it overlooks the number of records in the observation.

$$Mean\ Absolute\ Deviation\ (MAD) = \frac{\sum_{i=1}^{K} |P_o - P_e|}{K} \quad (5)$$

MAD is a measure of deviation of each figure from expected proportion. It takes into account.The absolute value of the difference between actual and expected proportion regardless of the value being positive or negative. The MAD critical values to check the conformity of data are given in Table 1.

DISCUSSION AND RESULTS

The data used for this study is taken from a sample of Hungarian Whole-Sale Trade sector. Like other sectors of economy, this sector also prone to misreporting of financial results. According to report of Association of Certified Fraud Examiners (ACFE, 2016), 41% of cases in Whole-sale trade sector are involved in corruption. For this study, a regional dataset of North plane of the country is considered for analysis. A total 2116 net sale digits were taken into consideration. An overall look of the figure 1 suggested that mostly of the digit frequencies are close to Benford's frequencies. First six digits are showing close conformity to expected frequencies. The larger digits show a diversion which is highest for digit nine. Higher digits show that actual frequencies of these digits are lower than expected (Figure 1)

For second digits analysis in figure 2, the digit three shows a highest negative deviation while digits five and eight show actual leads expected frequency. Rest all digits exhibit some level of conformity to Benford's line. The graph for first two digits shows some extreme deviation at points 48, 50, 70 and 96 (Figure 3). Rest all first two digits lie in an acceptable range.

To verify our null hypothesis: actual digit proportion equals Benford's proportion, Z- stat is applied. It confirms whether a particular digit from our data set is suspect (Durtschi et al., 2004). As a rule, if Z-stat value is 1.96 or higher, it shows p value of .05 (95 percent confidence). Z- Stat confirms only two of the first digit frequencies are significant. The digit eight and nine in the first digits test have Z- value greater than 1.96.(Nigrini, 1996). For second digits test, Z- stat for digit two is 2.427. All other frequencies show a mixed trend. For first two digits, four of the distributions shows a significant Z- value. Rest all shows insignificant behavior. Therefore our null hypothesis cannot be rejected in the basis of Z-Stat value.

For ignoring the size of the distribution, MAD test is applied. MAD test value gives us the difference between actual and expected proportion. Actual MAD result is then analyzed and compared with critical MAD values (Drake & Nigrini, 2000). MAD value for first digits is 0.00887. This shows the average deviation of actual frequency to Benford's line. Comparing critical MAD values in Table 1, the results show acceptable conformity of first digits. MAD results for second digits and first two digits have values (0.00555) and (0.00200) respectively which show that the deviation of frequency from expected frequency fall in the range of close to marginally acceptable conformity level (compared to critical values of MAD).

CONCLUSIONS

This study has applied Benford's law for analysis of digit pattern of accounting numbers. The main objective was to determine whether these digit patterns are showing conformity to Benford's law. For sake of this study, test sample is comprised of wholesale trade sector data of firms of Hungary. An overall analysis using Nigrini's template (Nigrini, 2012) suggested conformity to Benford's distribution. First digits analysis, second digits analysis and first two digit analysis is carried out in this study (Nigrini & Mittermaier, 1997). In order to check robustness of results, Z-stat is MAD test are used. Z-Stat suggests only non-conformed behavior of few (8 and 9) digit patterns. MAD value of fist digit, second digits and first two digits showed close to marginally acceptable conformity of digits.

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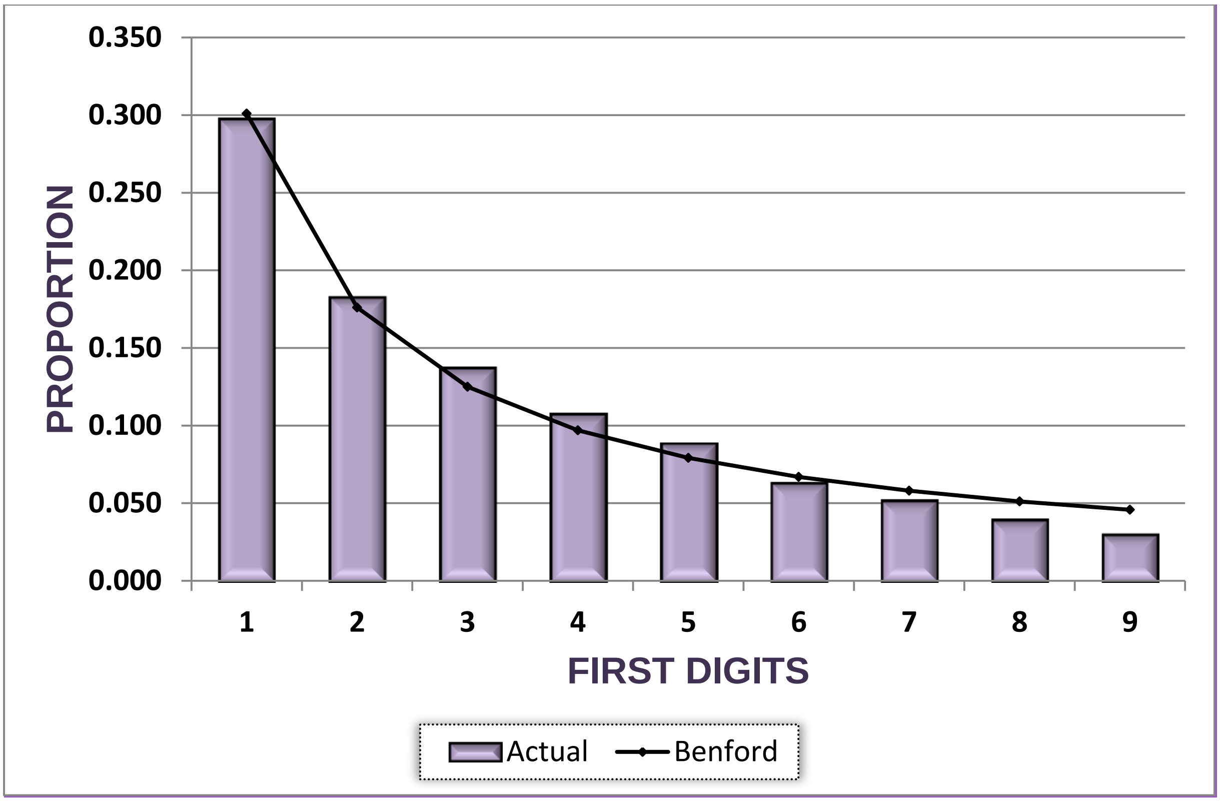
Table 1: Critical Values for Different MAD Values

Digits	Range	Results
First Digits	0.000 to 0.006	Close conformity
	0.006 to 0.012	Acceptable conformity
	0.012 to 0.015	Marginally acceptable conformity
	Above 0.015	Nonconformity
Second Digits	0.000 to 0.008	Close conformity
	0.008 to 0.010	Acceptable conformity
	0.010 to 0.012	Marginally acceptable conformity
	Above 0.012	Nonconformity
First-Two Digits	0.0000 to 0.0012	Close conformity
	0.0012 to 0.0018	Acceptable conformity
	0.0018 to 0.0022	Marginally acceptable conformity
	Above 0.0022	Nonconformity

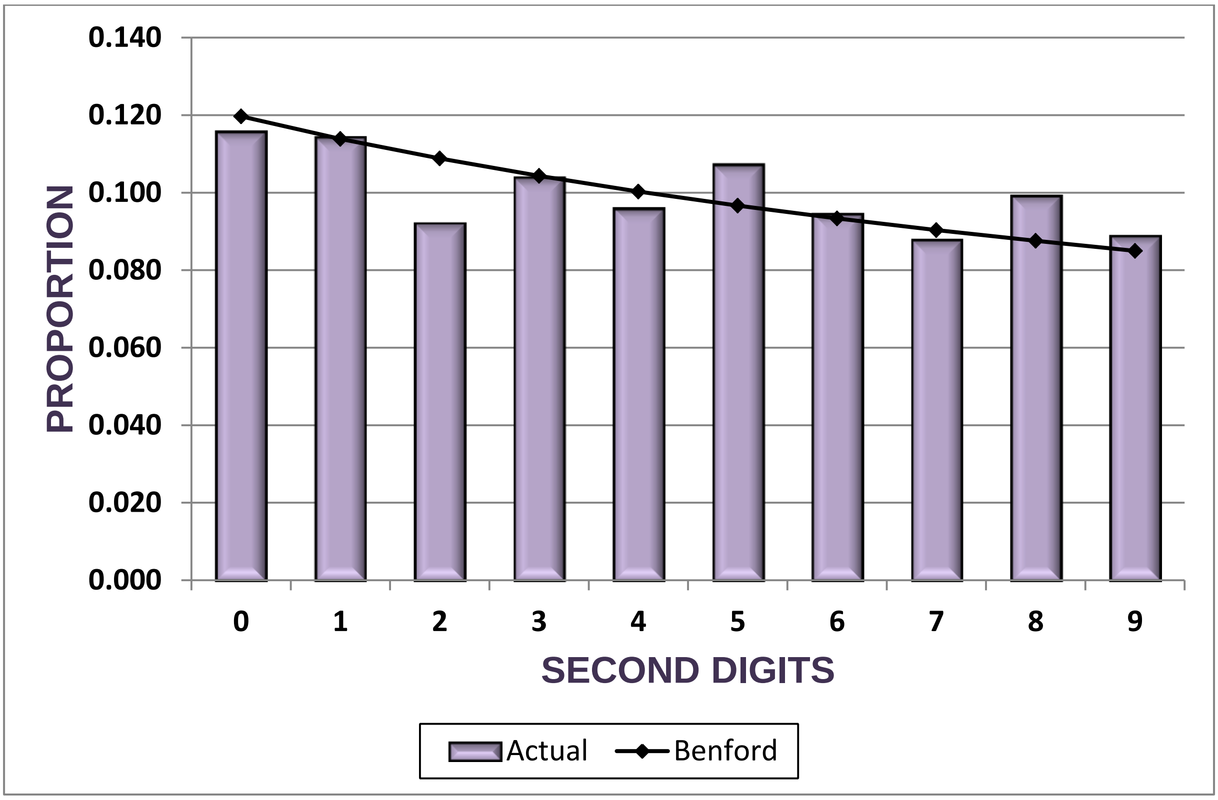
Source: (Nigrini, 2012)

Figure 1: First digits Analysis of Net Sales



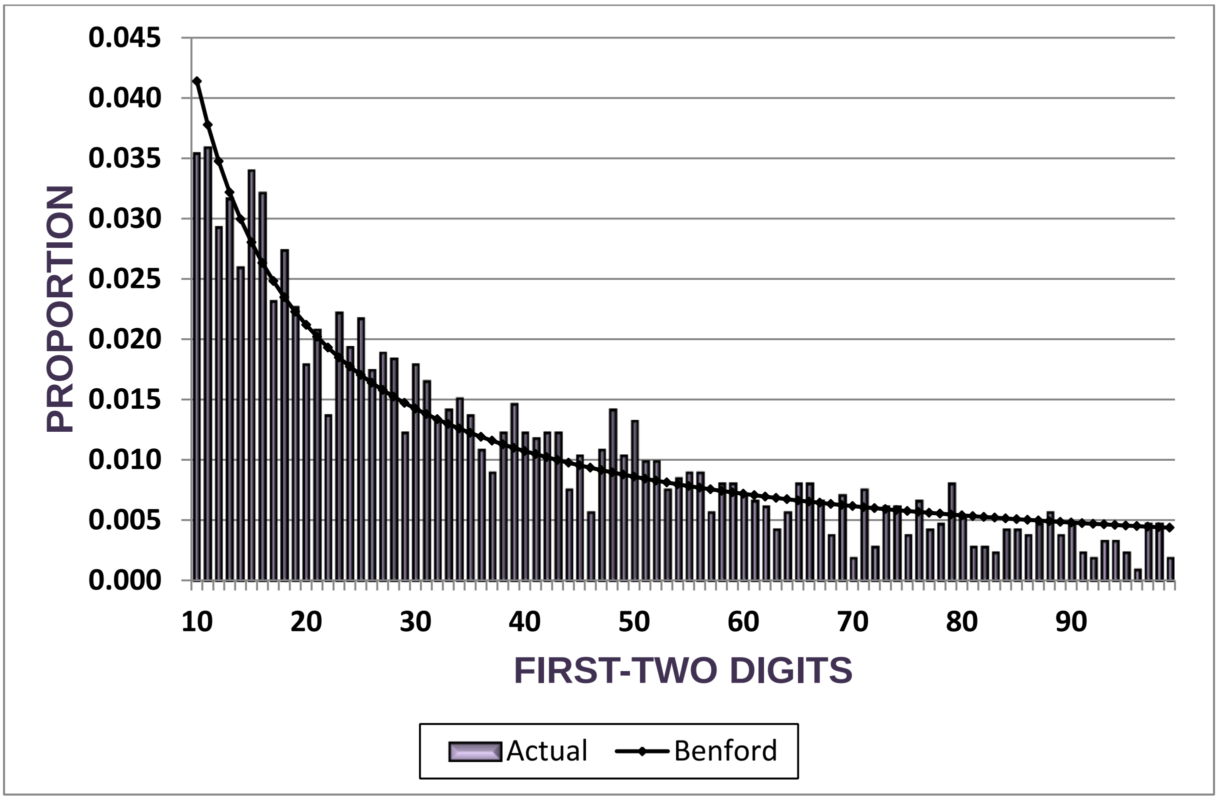
Source: Author’s Own Analysis of Net Sales in Hungarian Wholesale Trading Sector between 2011-2015

Figure 2: Second Digits Analysis of Net Sales



Source: Author's Own Analysis of Net Sales in Hungarian Wholesale Trading Sector between 2011-2015

Figure 3: First Two digits Analysis of Net Sales



Source: Author's Own Analysis of Net Sales in Hungarian Wholesale Trading Sector between 2011-2015